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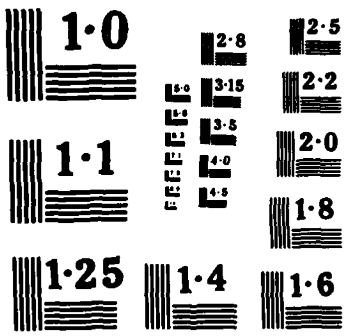

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DERIVING A UTILITY FUNCTION FOR THE ECONOMY

by

George B. Dantzig

TECHNICAL REPORT SOL 85-6

June 1985

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DERIVING A UTILITY FUNCTION FOR THE ECONOMY

George B. Dantzig

Abstract

The model we describe~~d~~ here has the same general features of the PILOT dynamic macro-economic model of U.S. designed to assess the long term impact of foreign competition, innovation, modernization, and energy needs. We derive the aggregate demand function of final consumer from individual demand functions in order to state its mathematical properties; we then estimate its parameters by a fit to empirical data. The equilibrium conditions are those of the Arrow-Debreu model, the only unusual feature is that investors calculate their rate of return using discounted normalized prices of future periods. If investors choose to normalize intra-period prices in the usual way by requiring that they sum to unity (or equivalently their average value is unity), the inverse demand functions turn out to be non-integrable. Equally satisfactory from the investors' point of view, is for them to choose instead to normalize intra-period prices by making these equal to $\bar{\pi}/(\bar{\pi}' H \bar{\pi})^{1/2}$ where H is a given positive-definite matrix and $\bar{\pi}$ is the vector of intra-period prices. In the latter case, ~~we show~~ It is shown that the inverse-demand functions are integrable and derive a utility function for the economy which if maximized subject to the physical-flow constraints implies the equilibrium conditions.

Keywords: economic growth,

DERIVING A UTILITY FUNCTION FOR THE ECONOMY

George B. Dantzig[1]

We begin by describing the structure of the $t = 1, \ldots, T$ time period model. It has the same general feature as the PILOT model of U.S. See references [12, 13].

<u>EQUILIBRIUM MODEL</u>. For $t = 1, \ldots, T$:

$$
\begin{array}{llcll}
 & \underline{\text{Cap. Req.}} & & \underline{\text{Cap. Avail.}} & \underline{\text{Dual Corresp.}} \\
(1) & +B_t Y_t & \le & +D_{t-1}Y_{t-1} + k_t & : \sigma_t \ge 0 \\
(2) & -A_t Y_t \quad + \quad X_t & \le & 0 & : \pi_t \ge 0 \\
 & -\underline{\text{Product.}} \quad + \quad \underline{\text{Consump.}} & & & \\
 & \underline{\text{Investor rate of return}} & & -\underline{\text{Endow. Value}} & \\
(3) & -B_t^T\sigma_t + A_t^T\pi_t & \le & -D_t^T\sigma_{t+1} & : Y_t \ge 0 \\
(4) & -\pi_t \quad + \delta^{-t+1}F_t(X_t) & \le & 0 & : X_t \ge 0 \\
 & \underline{\text{Inverse demand function}} & & & \\
\end{array}
$$

$$(5) \qquad \hat{\sigma}_t\sigma_t = 0, \qquad \hat{\pi}_t\pi_t = 0, \qquad \hat{Y}_t Y_y = 0, \qquad \hat{X}_t X_t = 0$$

where variables with a hat over them are resp. the slack vectors for the inequalities (1), ..., (4) and δ^{-t+1} is the discount factor. Relations (5) are the <u>complementary-slackness</u> conditions. See references [1,4,14].

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[1] Based on joint work with P.H. McAllister and J.C. Stone.

The matrix structure is skew symmetric except for the term in lower right diagonal. It is of interest to note that if the economy were driven by a utility function of the form $U = \Sigma\ \delta^{-t+1}\ U_t(X_t)$, the Kuhn-Tucker conditions derived by maximizing U subject to primal physical-flow conditions (1) and (2) would have exactly this structure where $\partial U_t/\partial X_t = F_t(X_t)$. If the latter condtions hold, we say the vector functions $F_t(X_t)$ in the model are <u>integrable</u>. See references [3,10].

(1) states that the capacity requirements to meet production and investment levels Y_t in period t must not exceed capacity carried down from period $t-1$ plus any exogeneous capacity supplied k_t. For period 1, the term $+D_0Y_0$ is omitted; k_1 is the initial capacity (endowment) vector.

(2) states that the consumption vector X_t of the final consumers must not exceed the net-output of production after investment. The consumption vector of government services is treated as part of A_tY_t and is not shown separately.

(3) states, in case of an intra-period <u>production</u> activity j, that production level $Y_t(j)$ will rise to the point of non-profitability and if strictly non-profitable will not be used because of the complementary slackness condition $\bar{Y}_tY_t = 0$. In the case of an inter-period <u>investment</u> activity, the investor must receive his discounted rate of return δ or he won't invest. Prices π_t are <u>normalized</u> intra-period t prices <u>discounted</u> by δ^{-t+1}. On the right hand side, $D_t^T\sigma_{t+1}$ is the discounted

value of future capacity (endowments). For $t = T$ this term is set $= 0$. Note: Undiscounted prices will be denoted by $\bar{\pi}_t$.

(4) is an equation for $X_t > 0$. It relates prices to consumption. The direct demand function is an expression of the form $X_t = \mathcal{F}(\pi_t, \pi_t' X_t)$ where $\mathcal{F}$ is a homogeneous function in π_t of degree 0 and $\pi_t' X_t$ is the attained level of aggregate income I_t. It follows from the homogenity of $\mathcal{F}$ that the inverse demand function $F(X)$ (expressing π_t as a function of X_t) can only determine prices within a scale factor. It is the freedom to select the scale factor which allows us to choose the formula for normalizing intra-period prices $\bar{\pi}_t$. We define $F(X)$ as equal to normalized intra-period prices.

We assume that the investors will want to use discounted normalized intra-period t prices in calculating their rate of return. If intra-period prices $\bar{\pi}_t$ are normalized by scaling them so that their average is unity then the average of the discounted prices π_t is δ^{-t+1}. Since $\pi_t = \delta^{-t+1} F_t(X_t)$, when $X_t > 0$, this implies that F_t satisfies $eF_t(X_t) \equiv 1$ for all choices of X_t where $e = (1/n, \ldots, 1/n)$ and n is the number of components of X_t.

If this usual way of normalizing intra-period prices is used, i.e., so that their average is $1/n$ or as we prefer 1, it will turn out, however, that the equilibrium problem (1), ..., (5) is non-integrable, i.e., there does not exist a utility function which if maximized subject to physical-flow conditions (1) and (2) and $(X_t, Y_t) \geq 0$ yields the equilibrium solution. We will show, on the other hand, that there is another way to do

the normalization, equally satisfactory from the view point of the investor, that is <u>integrable</u> and from which it is easy to derive an aggregate utility function for the economy.

<u>Derivation of the aggregate demand function</u>

Let I_t be the <u>value of endowments</u> used to produce the consumption vector X_t in period t. In terms of the prices of the model, the <u>value of endowments</u> available to period t is $\sigma_t^T(D_{t-1}Y_{t-1} + k_t) = \sigma_t^T B_t Y_t$ and the value passed down to period $t+1$ is $\sigma_{t+1}^T D_t Y_t$. Their difference is I_t by definition. By (1), (2), and (5), it is clear $I_t = \pi_t X_t$, the <u>aggregate take-home income</u>. We will often omit the subscript t to simplify the notation; thus we write $I = I_t$.

We will use the index i to denote the <u>i-th consumer</u>; $I^i = I_t^i$ to denote his personal income in period t; and $X^i = X_t^i$ to denote his consumption vector. Let $U^i(X^i)$ denote his utility function, and let α_i be his <u>assumed constant share</u> of aggregate income I. The price vector corresponding to the components of X_t^i is denoted by $\pi = \pi_t$.

The budget constraints are

$$\text{(6)} \qquad \begin{aligned} \pi' X^i &= \alpha_i I \; ; \\ \Sigma\, \pi' X^i &= \; I \, , \qquad \text{where } \Sigma\alpha_i = 1. \end{aligned}$$

At equilibrium we have for individual i,

$$\text{(7)} \qquad \partial[U^i(X^i) - \lambda(\pi' X^i)] \,/\, \partial X^i = 0 \, ,$$

where the Lagrange multiplier λ is chosen so that his budget constraint $\pi' X^i = \alpha_i I$ is satisfied.

We approximate $U^i(X^i)$ in the neighborhood of an equilibrium solution by a general quadratic function of the form

$$(8) \qquad U^i(X^i) \doteq (M^i S^i)^T X^i - (1/2)\,(X^i)^T M^i (X^i) + \text{Constant}_i \;,$$

where M^i in general is a symmetric non-singular matrix. We <u>assume</u> M^i <u>to be positive definite</u> and S^i to be a <u>fixed vector</u> such that $X^i << S^i$ for any X^i attainable by the model. Of course $S^i = S^i_t$ is strictly positive; it could be different for different t. In the PILOT model $S_t = \sum_i S^i_t$ grows proportional to population size, i.e., as the number of individuals i grows.

We substitute the approximation (8) into (7) obtaining

$$(9) \qquad M^i(S^i - X^i) \doteq \lambda \cdot \pi \;,$$

$$(10) \qquad S^i - X^i \doteq \lambda \cdot H^i \pi \;, \qquad \text{where } H^i = (M^i)^{-1} \;.$$

Note that $H^i = (M^i)^{-1}$ is also symmetric and positive definite.

We can now use the budget constraint to determine λ. Multiplying (10) by π' on the left, setting $\pi' X^i = \alpha_i \mathbf{I}$, we can solve for λ and substitute into (9). This yields the <u>local</u> approximation for the <u>demand function of individual i</u> in the neighborhood of the equilibrium solution as a function of prices and aggregate income $\mathbf{I}$:

$$(11) \qquad S^i - X^i \doteq \left(\frac{\pi' S^i - \alpha_i \mathbf{I}}{\pi' H^i \pi}\right) H^i \pi \;.$$

Summing over all i and setting $S = \Sigma S^i$ and $X = \Sigma X^i$ yields the local approximation to the aggregate demand function.

$$(12) \qquad S - X \doteq [\sum_i \frac{\pi'(S^i - \alpha_i S)}{\pi' H^i \pi} H^i]\pi + (\pi' S - I) \cdot [\sum_i \frac{\alpha_i}{\pi' H^i \pi} H^i]\pi \quad .$$

Denoting the bracket terms above by $\bar{G}$ and $\bar{H}$, we have

$$(13) \qquad S - X \doteq \bar{G}\pi + (\pi' S - I) \cdot \bar{H}\pi \ .$$

Note $\bar{G}$, $\bar{H}$ are square symmetric matrices because they are weighted sums of square symmetric matrices H^i whose elements are functions of π only. It is easy to verify that $\bar{G}$, $\bar{H}$ have the following properties:

$$(14) \qquad \pi' \bar{G} \pi \equiv 0 \ , \qquad \pi' \bar{H} \pi \equiv 1$$

for all π. Moreover, $\bar{H}$ is positive definite because it is a non-negative sum of positive-definite symmetric matrices H^i. The individual demand functions (11) and the aggregate demand function (13), for fixed prices π, are locally linear in $I^i = \alpha_i I$ and I resp.

We now make a fit to empirical data to see if the local linear approximations can be extended to a broad range of I. For this purpose we will need survey data of personal consumption as a function of take-home income at fixed prices and we will need the distribution of take-home income α_i. See references [2, 7, 8].

Personal consumption

At fixed survey year prices $\bar{\pi}_0 = (\bar{\pi}_{01}, \ldots \bar{\pi}_{0k}, \ldots \bar{\pi}_{0n})$, personal consumption $C_k(I^i)$ during the survey year of item k, say food measured in physical units, is known from survey data as a function of personal income I^i, see Figure 1:

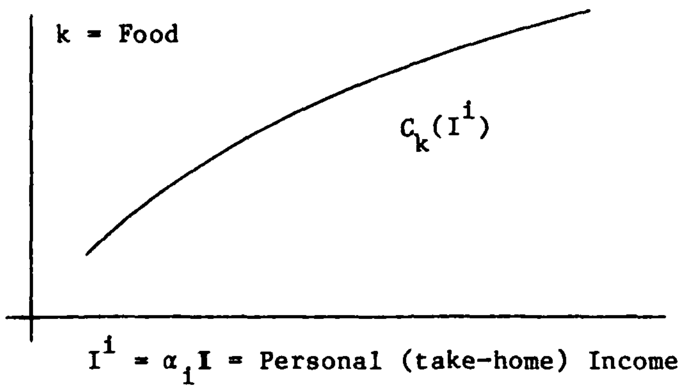
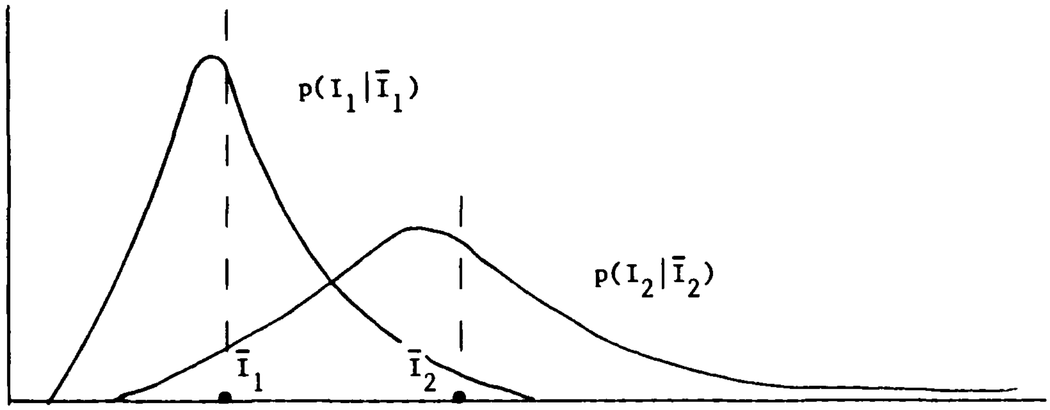


FIGURE 1: Consumption of Food as a Function of Personal Take-home Income (prices are fixed at $\bar{\pi} = \bar{\pi}_0$))

Since the sum of the consumption of item k weighted by prices $\bar{\pi}_{0k} \geq 0$ over all items adds up to take-home income, it follows that if some curves, like food, display a decreasing slope with increasing income I^i then others must show an increasing slope with increasing income. In PILOT, expenditure patterns $C_k(I^i)$ of individuals for some future period t at constant prices $\bar{\pi}_0$ are assumed to be the same as the survey year with some adjustments to reflect any known trends in "taste".

Take-home income

The distribution of take-home income I^i expressed in base-year dollars has changed over the years in U.S. but generally retains the same shape except it spreads out proportionally as per-capita take-home income $\bar{I}$ increases. That is to say, the proportional share of endowments α_i has remained more or less constant over the years, [8,2].



I = Personal (take-home) Income

FIGURE 2: Distribution of Personal Income $I^i = I_1$ and I_2 when average income is $\bar{I}_1$ and $\bar{I}_2$ resp.

Letting $p(I|\bar{I})$ denote the distribution of personal take-home income when per capita income is $\bar{I}$, we are assuming that $p(I|\bar{I})$ can be derived from $p(I|\bar{I}_0)$ by

$$p(I|\bar{I}) = \theta \cdot p(\theta I|\bar{I}_0) \tag{15}$$

where $\theta = \bar{I}_0/\bar{I}$.

Per-capita consumption

At fixed survey-year prices $\bar{\pi}_0$, average (per capita) consumption of item k as a function of average income $\bar{I}$ can now be derived from personal consumption curves $C_k(I^i)$ and the distribution of income $p(I^i|\bar{I})$ by convolution:

$$(16) \qquad \bar{X}_{tk}(\bar{I}) = \int_{I=0}^{\infty} p(I|\bar{I})\, C_k(I)\, dI$$

$$= \theta \int_{I=0}^{\infty} p(\theta I|\bar{I}_0)\, C_k(I)\, dI\ , \qquad \theta = \bar{I}_0/\bar{I}\ .$$

Note that $\bar{X}_{tk}(\bar{I})$ is per-capita consumption as a function of per-capita income $\bar{I}$ expressed in survey-year dollars and assumes prices are fixed at $\bar{\pi}_0$. Period t prices, expressed in survey year dollars, may differ from $\bar{\pi}_0$. Later on we derive how it varies with $\bar{\pi}_t$.

At fixed survey-year prices $\bar{\pi}_0$, using personal consumption data as a function of take-home income and the observed distribution of income, the functions $\bar{X}_{tk}(\bar{I})$ have been computed in the manner described for over a hundred commodities by M. Avriel and for aggregated key commodities (using more recent data) by P.H. McAllister. These functions turn out to be remarkably linear. See Figure 3 and references [7,2,8].

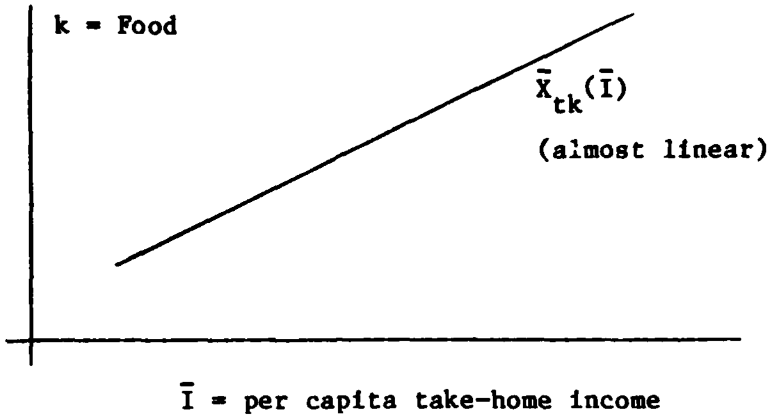


FIGURE 3: Consumption of Food as a Function of Per Capita Take-home Income (prices are fixed at $\pi_t = \pi_0$).

Global versus Local Fit

At fixed survey year prices $\bar{\pi}_0$, individual demand functions do not appear to be linear with I^i, see Figure 1. However, we obtain an excellent global fit to the aggregate demand functions using a linear function of per capita income $\bar{I}$, see Figure 3; or multiplying $\bar{I}$ by population size to obtain $\mathbf{I}$, by a linear function in $\mathbf{I}$. Therefore we accept (13) as the form of our global fit to the aggregate demand function. We think of $S = S_t$ as a kind of "satiation" vector many times larger than any $X = X_t$ attained in any period.

Returning now to our aggregate demand function (13), we now postulate that as aggregate income $\mathbf{I}$ approaches $\pi'S$, the income sufficient to purchase the satiation vector S, aggregate consumption X tends to S. We are assuming:

$$(17) \qquad X \to S \quad \text{as} \quad \mathbf{I} = \pi'X \to \pi'S \; .$$

In (13), if we fix prices π and let $I \to \pi'S$, we observe that $\bar{G}\pi \equiv 0$ for all π so that we can drop the first term of (13). Now $\bar{H}$, in the second term, is a symmetric positive-definite matrix whose elements depend on π with the property that $\pi'\bar{H}\pi \equiv 1$ for all π; it follows that the general form of $\bar{H}$ is

$$\bar{H} = \frac{1}{\pi' H \pi} H \tag{18}$$

where H is a positive-definite matrix whose elements can depend on π.

Our global fit[2] to the aggregate demand function thus reduces to

$$S - X = \frac{(\pi' S - I)}{\pi' H \pi} \cdot H\pi \,, \qquad X << S \,. \tag{19}$$

For purposes of estimation of S and H, we assume H to be a constant matrix as well as symmetric and positive definite.

To obtain the inverse demand function $F(X)$ that expresses π in terms of X, we solve (19) for π. Since $I = \pi'X$, the right-hand side is a homogeneous function in π of degree 0 implying that π can only be determined within a scale factor. Clearly π is proportional $M(S-X)$ where $M = H^{-1}$ and we are free to choose the proportionality factor so that the prices of the model are automatically normalized before discounting. In the model relation (4), $\pi = \delta^{-t+1} F(X)$ when $X > 0$ so that $F(X)$ denotes normalized intra-period prices.

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[2] McAllister has estimated S and H in (19) using 22 years of empirical data of per-capita consumption patterns, per-capita take-home income, and prices under the assumption that H is a constant matrix that is symmetric and positive definite. See reference [3].

If investors calculate their rate of return based on intra-period prices $\bar{\pi}_t$ normalized by $\bar{\pi}_t / e\bar{\pi}_t$ where $e = (1/n, \ldots, 1/n)$, then the scale factor is chosen so that normalized intra-period prices $F(X)$ satisfy

$$(20) \qquad F(X) = \frac{1}{eM(S-X)} \cdot M(S-X) , \qquad M = H^{-1} .$$

Note $eF(X) \equiv 1$ for all X and $F(X)$ does not depend on the scaling of M. Note that when prices $\bar{\pi}_t^* = (1, 1, \ldots, 1)$ that $(1, 1, \ldots, 1)$ are normalized prices.

However, if investors calculate their rate of return based on intra-period prices $\bar{\pi}_t$ normalized by $\bar{\pi}_t/(\bar{\pi}_t' \, H \, \bar{\pi}_t)^{1/2}$, the scale factor is chosen so that normalized intra-period prices $F(X)$ satisfy

$$(21) \qquad F(X) = \frac{1}{[(S-X)^T \, M(S-X)]^{1/2}} \cdot M(S-X) , \qquad M = H^{-1} .$$

Note that $[F(X)^T \, H \, F(X)]^{1/2} \equiv 1$ for all X but that $F(X)$ does depend on scaling of M. We can rescale H and hence M in (21) so that if intra-period prices $\bar{\pi}_t^* = (1, \ldots, 1)$, they will also be $(1, \ldots, 1)$ after normalization, i.e., satisfy $[\bar{\pi}_t^* \, H \, \bar{\pi}_t^*] = 1$. Therefore H is rescaled so that

$$(22) \qquad \text{Rescaled } H = H/(\pi^* \, H \, \pi^*)$$

and $M = [\text{Rescaled } H]^{-1}$ is then used in (21).

Both ways to normalize prices appear to be equally satisfactory from the view point of the investor figuring his rate of return. However if (20) is used, we will give an easy proof below that no utility function for the economy exists; whereas the interesting thing is that if (21) is used, there is one.

Proof: Assume, <u>on the contrary</u>, that a utility function does exist for (20). Consider a one period model so that we are maximizing the utility $U(X)$ subject to the primal constraints (1) and (2). Further suppose X_1 has only two components so that $X_1 = (X_{11}, X_{12})$. Let $S_1 = (S_{11}, S_{12})$, $S_1 - X_1 = (S_{11} - X_{11}, S_{12} - X_{12})$. Let $\pi_1 = (\pi_{11}, \pi_{12})$. $M = [m_{ij}]$ is a 2×2 symmetric non-singular matrix. Let

$$(23) \qquad V_1 = S_{11} - X_{11} , \quad V_2 = S_{12} - X_{12} , \quad V = (V_1, V_2) .$$

At a maximum the Kuhn-Tucker conditions $\partial U/\partial X = \pi$ hold. Since $X = S-V$, we have from (20)

$$(24) \qquad \partial U/\partial X_{11} = \pi_{11} = (m_{11}V_1 + m_{12}V_2)/D ,$$

$$(25) \qquad \partial U/\partial X_{12} = \pi_{11} = (m_{12}V_1 + m_{22}V_2)/D ,$$

where the denominator $D = (m_{11}+m_{12})V_1 + (m_{12}+m_{22})V_2$.

In order for a utility function to exist, the second partial $\partial^2 U/\partial X_{11}\partial X_{12}$ computed from (24) should agree with $\partial^2 U/\partial X_{12}\partial X_{11}$ computed from (25) <u>for all choices of</u> X_{11}, X_{12}. Setting these 2nd partials equal to each other, we obtain

$$(26) \qquad -\frac{12}{D} + \frac{m_{11}V_1 + m_{12}V_2)\ (m_{12} + m_{22})}{D^2}$$

$$\equiv -\frac{m_{12}}{D} + \frac{(m_{12}V_1 + m_{22}V_2)(m_{11} + m_{12})}{D^2}$$

which reduces to $(m_{11}m_{22} - m_{12}^2)\ (V_1 - V_2)/D^2 \equiv 0$, which does not hold for all choices of (X_{11}, X_{12}) because $(V_1-V_2) = (S_{11}-X_{11})-(S_{12}-X_{12})$, a contradiction.

On the other hand, if we normalize intra-period prices $\bar{\pi}$ by $\bar{\pi}/(\bar{\pi}'H\bar{\pi})^{1/2}$, then the equilibrium problem (1), ..., (5) is equivalent to solving the convex-programming problem:

MATHEMATICAL PROGRAMMING MODEL.

Find Minimum $-U(X)$, $(X_t, Y_t) \geq 0$:

$$(27) \qquad -U(X) = \sum_{t=1}^{T} \delta^{-t+1}[(S_t-X_t)^T M_t(S_t - X_t)]^{1/2}$$

subject to primal flow conditions for $t = 1, \ldots, T$:

				Dual Corresp.
(28)	$+B_tY_t$		$\leq + D_{t-1}Y_{t-1} + k_t$	$: \sigma_t \geq 0$
	$-A_tY_t$	$+X_t$	≤ 0	$: \pi_t \geq 0$

Because M_t are positive definite matrices, it is not difficult to prove each term of (27) is a convex function in X_t and hence their sum $-U$ is also. References [5, 6, 9, 11] discuss the existence of solutions for convex programs and the techniques for their solution. The primal and

dual variables of the optimum solution satisfy the Kuhn-Tucker conditions which are precisely those of the equilibrium problem (1), ..., (5).

We conclude that the economy will grow if it has the technology and initial endowments to grow and if it pays to trade off movement of the consumption vector X_t away from the "satiation" vector S_t in earlier periods for considerably larger movements towards the satiation in later periods where the measure of disutility function for period t is given by

$$(29) \qquad -U_t(X_t) = \delta^{-t+1}[(S_t-X_t)^T M_t(S_t-X_t)]^{1/2} .$$

By (4) and (21), $-U_t(X_t) = \pi_t'(S_t-X_t)$ so that the disutility is the discounted <u>additional</u> aggregate income needed to purchase the "satiation" vector — i.e., the more additional income required the lower the "standard of living".

REFERENCES

[1] Arrow, K., and G. Debreu: "Existence of an Equilibrium for Competitive Economy," Econometrica, 22 (1954), 265-290.

[2] Avriel, M.: "Modeling Personal Consumption of Goods in the PILOT Model," Technical Report SOL 76-17, Systems Optimization Laboratory, Department of Operations Research, Stanford University, August 1976. Recent updates by P.H. McAllister, based on later survey data, confirm the earlier study.

[3] Cottle, R.W., and G.B. Dantzig: "Complementary Pivot Theory of Mathematical Programming," Linear Algebra and Its Applications, 1 (1968), 103-125.

[4] Debreu, G.: Theory of Value, Yale University Press.

[5] Eaves, B.C.: "Homotopies for the Computation of Fixed Points," Math. Programming, 3 (1972), 1-22.

[6] Garcia, C.B., and W.I. Zangwill: Pathways to Solutions, Fixed Points and Equilibria, Prentice Hall, 1981, p. 402.

[7] Gorman, W.M.: "Community Preference Fields," Econometrica, 21 (1952), 63-80.

[8] Iusem, Alfredo: "Utility Functions with Affine Demand Curves," in PILOT-1980 Energy-Economic Model, Model Description, Electric Power Institute, Vol. 1. Project RP652-1, November 1981.

[9] Kakutani, S.: "A Generalization of Brouwer's Fixed-Point Theorem", Duke Mathematical Journal, 8 (1941), 457-459.

[10] Kuhn, H.W. and A.W. Tucker, "Non-linear Programming," Econometrica, 19 (January 1951), 50-51 (abstract).

[11] Manne, A.S.: "Survey on Computable Equilibrium Models," Operations Research Department, Stanford University, August 1984.

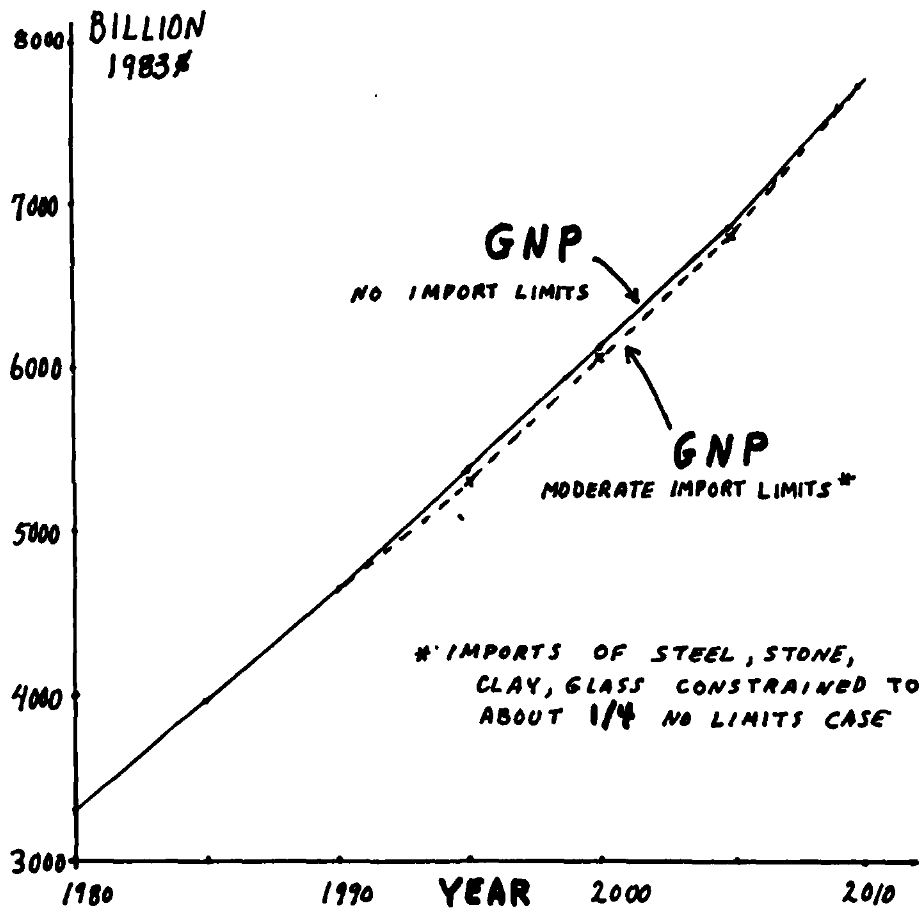
[12] McAllister, P.H., and J.C. Stone: "Will Planned Electricity Capacity Limit Economic Growth," Technical Report SOL 84-1, Systems Optimization Laboratory, Department of Operations Research, Stanford University, March 1984.

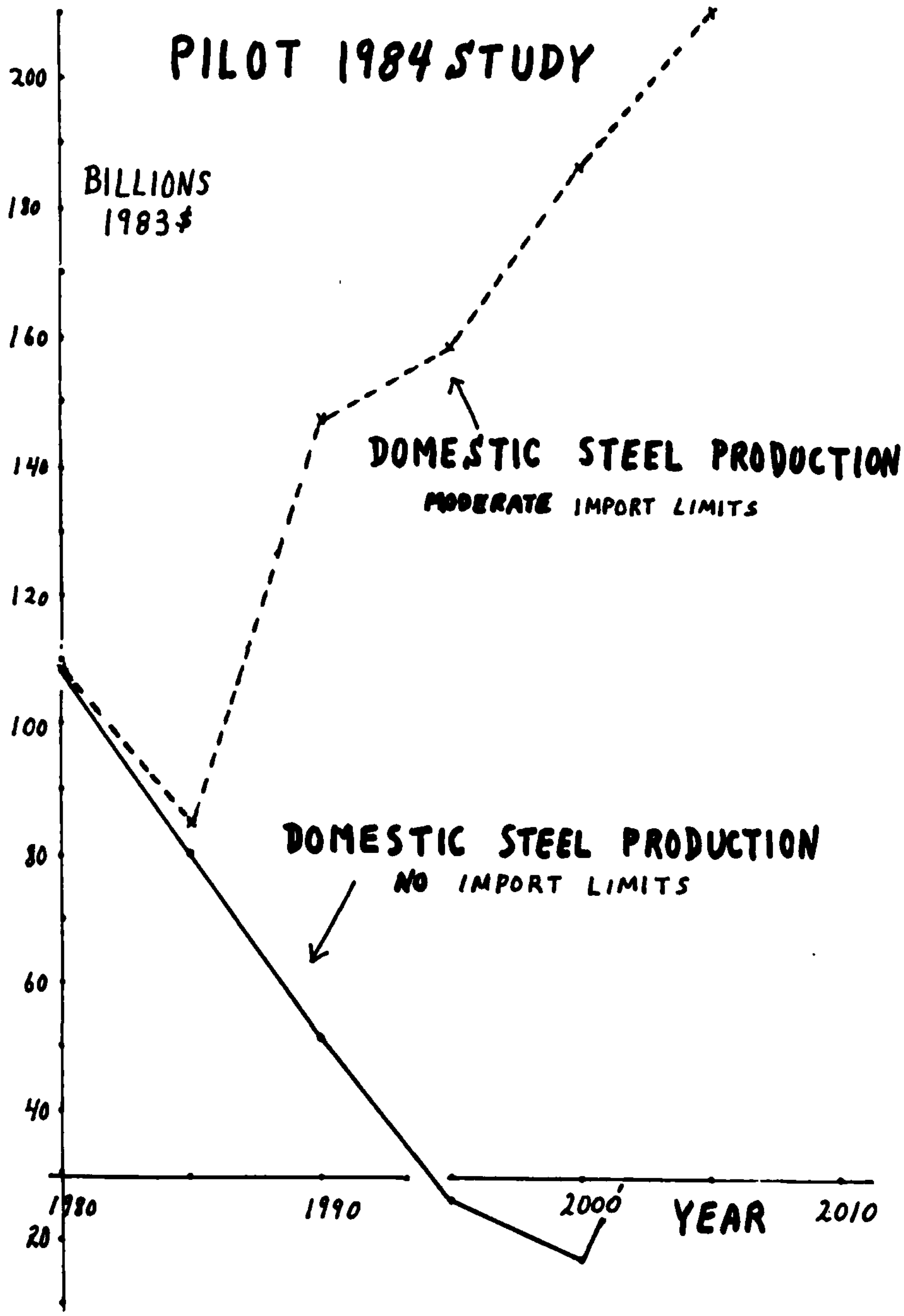
[13] PILOT-1980 Energy-Economic Model, Model Description, Electric Power Institute, Vol. 1. Project RP652-1, November 1981.

[14] Scarf, H.: "On the Computation of Economic Equilibrium Prices," in Ten Economic Studies in the Tradition of Irving Fisher, New York: John Wiley, 1967.

PILOT 1984 STUDY

(Non-linear Utility Function Maximized)




PILOT 1984 STUDY
BILLIONS
1983$
200
180
160
140
120
100
80
60
40
20
1980
1990
2000
2010
YEAR
DOMESTIC STEEL PRODUCTION
MODERATE IMPORT LIMITS
DOMESTIC STEEL PRODUCTION
NO IMPORT LIMITS

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dynamic equilibrium	fixed-point computation

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TR SOL 85-6: DERIVING A UTILITY FUNCTION FOR THE ECONOMY
by George B. Dantzig

The model we describe has the same general features of the PILOT dynamic macro-economic model of U.S. designed to assess the long term impact of foreign competition, innovation, modernization, and energy needs. We derive the aggregate demand function of final consumer from individual demand functions in order to state its mathematical properties; we then estimate its parameters by a fit to empirical data. The equilibrium conditions are those of the Arrow-Debreu model, the only unusual feature is that investors calculate their rate of return using discounted normalized prices of future periods. If investors choose to normalize intra-period prices in the usual way by requiring that they sum to unity (or equivalently their average value is unity), the inverse demand functions turn out to be non-integrable. Equally satisfactory from the investors' point of view, is for them to choose instead to normalize intra-period prices by making these equal to $\bar{\pi}/(\bar{\pi}'H\bar{\pi})^{1/2}$ where H is a given positive-definite matrix and $\bar{\pi}$ is the vector of intra-period prices. In the latter case, we show that the inverse-demand functions are integrable and derive a utility function for the economy which if maximized subject to the physical-flow constraints implies the equilibrium conditions.

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